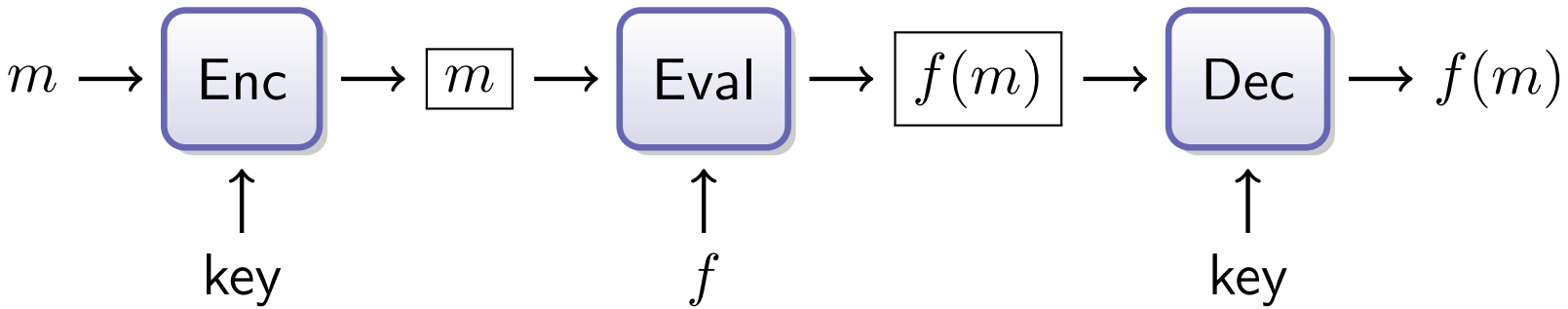
New Directions in Fully Homomorphic Encryption

Chris Peikert

University of Michigan

SAC Stafford Tavares Lecture, 26 Aug 2026

Fully Homomorphic Encryption [RAD'78,Gentry'09,...]



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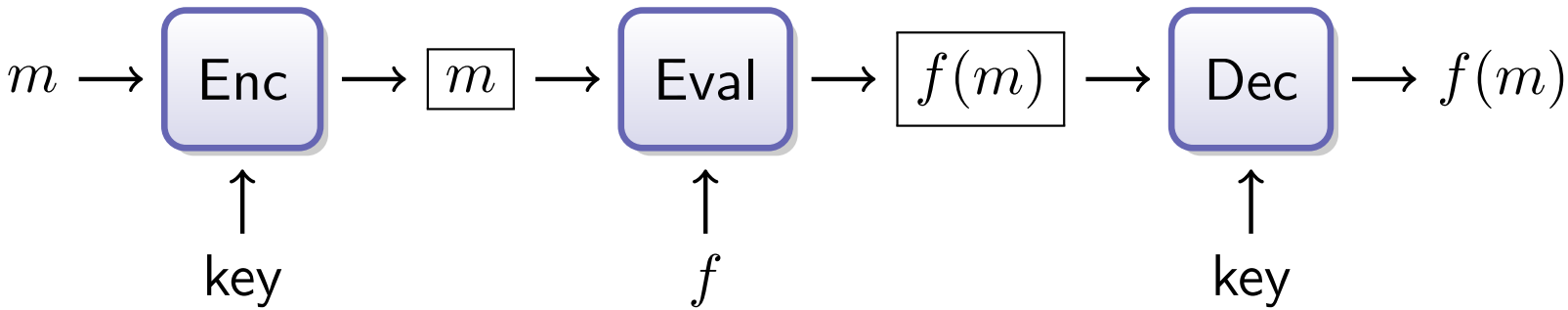

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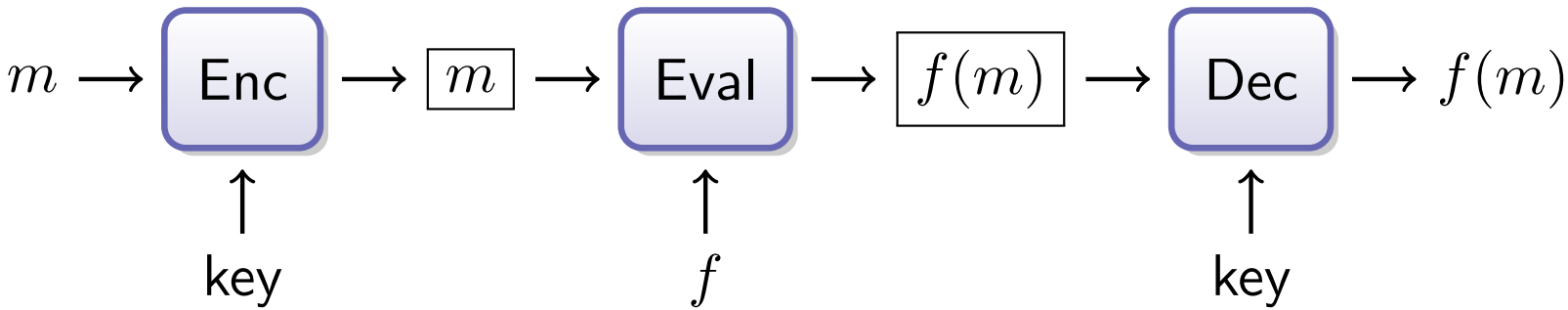

- Compute on data without seeing it
- For exact arithmetic, plaintexts live in a finite ring like $\mathbb{Z}_p$
- Modern schemes are highly capable, but have two major pain points

Two Pain Points for Exact-Arithmetic FHE

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Two New Directions

High precision via decomposition [Peikert–Zarchy–Zyskind, CRYPTO'26]

- Simple, generic technique for arbitrary modulus and ring (or no ring!)
- Parameters can match classic FHE for exponentially smaller modulus
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Flexible SIMD packing via cyclotomic subfields [Peikert–Pepin, TCC'25]

- Get any desired slot type, w/ many tensor shapes and security levels
- Retain fast arithmetic, homomorphic linear algebra, and packed bootstrapping

Direction I: High-Precision Plaintexts

Decompose plaintexts, but not the computation

(Peikert–Zarchy–Zyskind ePrint 2025/2321)

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For $p \geq 2^{64}$, this is a substantial concrete cost!

A Fragmented Landscape

[GC'14,CLPX'18,GV'25,BK'25,K'25,K'26,CPL'26,BFGGMS'26,...]

Approach	**Main advantage**	**Main compromise(s)**
Bitwise TFHE	Arbitrary p	$\Omega(\log^2 p)$ mult, slow add
Special modulus ideals	Simple, fast	Rare moduli, coupled rings
CKKS-based methods	Arbitrary p	Complex, mult $\supseteq$ bootstr
Special non-cyclo ring	Fast for $p = 2^k$	no SIMD, slow bootstrap

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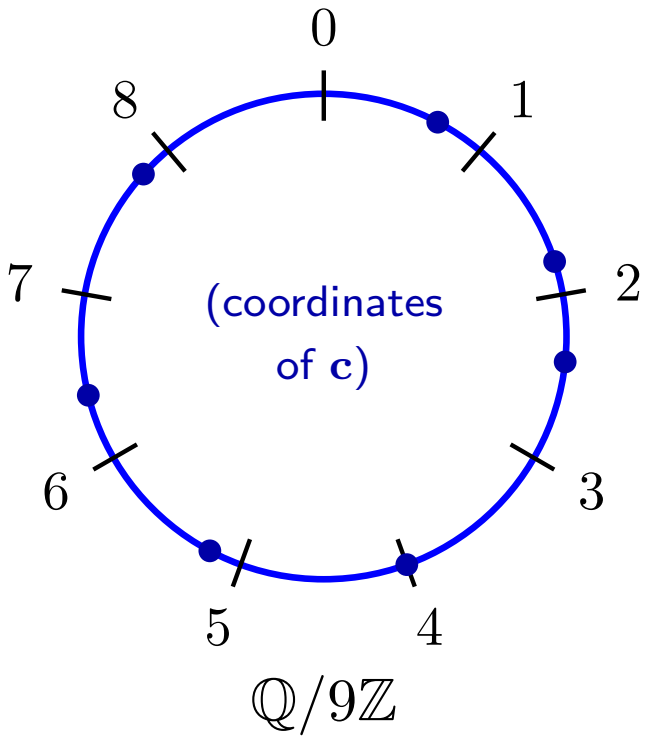
- **Key Idea:** Replace $\mathbb{Z}_p$ with an isomorphic "higher-dimensional" presentation having much smaller representatives

Warmup: Simplified BFV at the Right Scale



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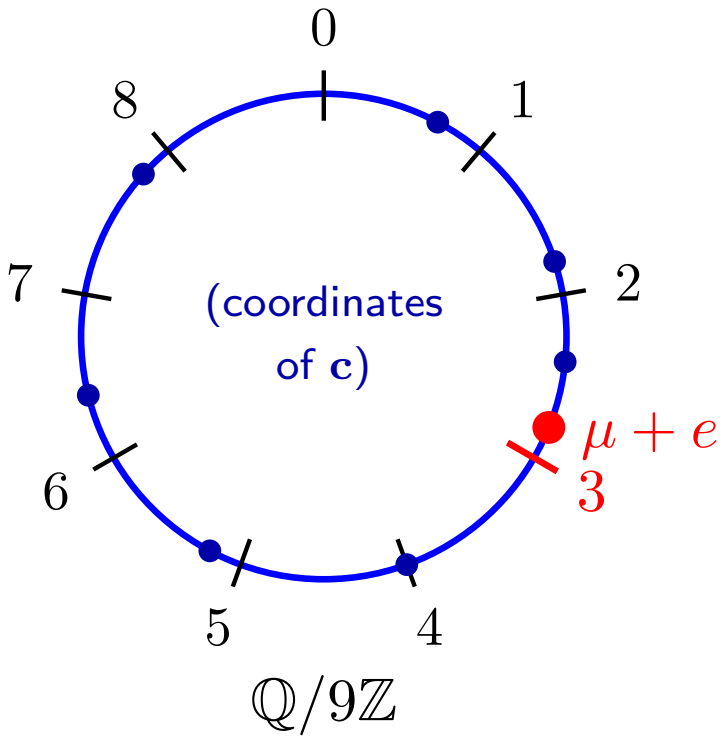

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Evaluate at the short secret $\mathbf{s} \in \mathbb{Z}^n$, then round to the nearest integer modulo p.

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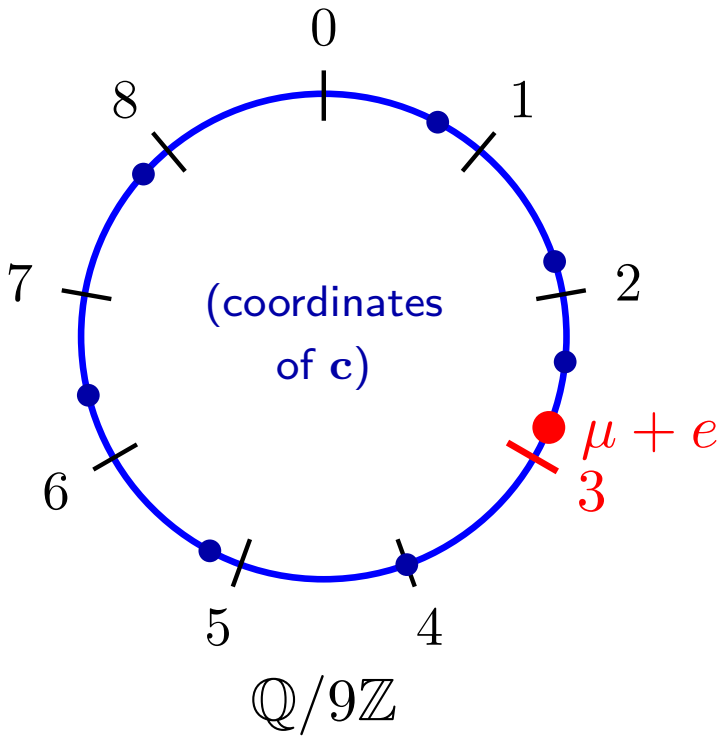

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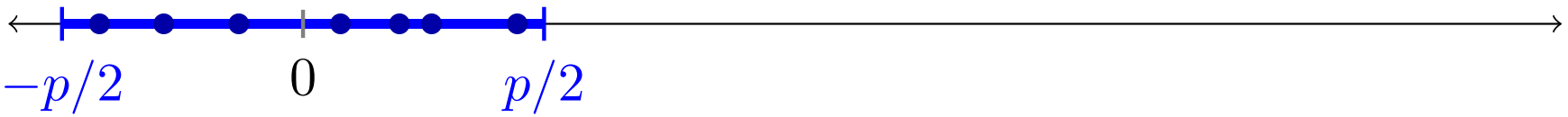
lifted ciphertext coordinates $\widehat{c_i} \in [-p/2, p/2)$:



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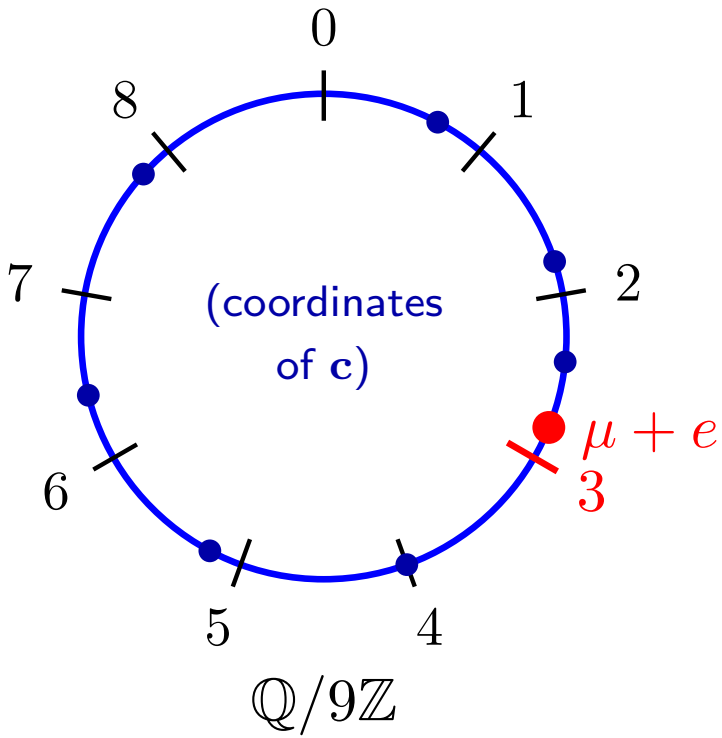

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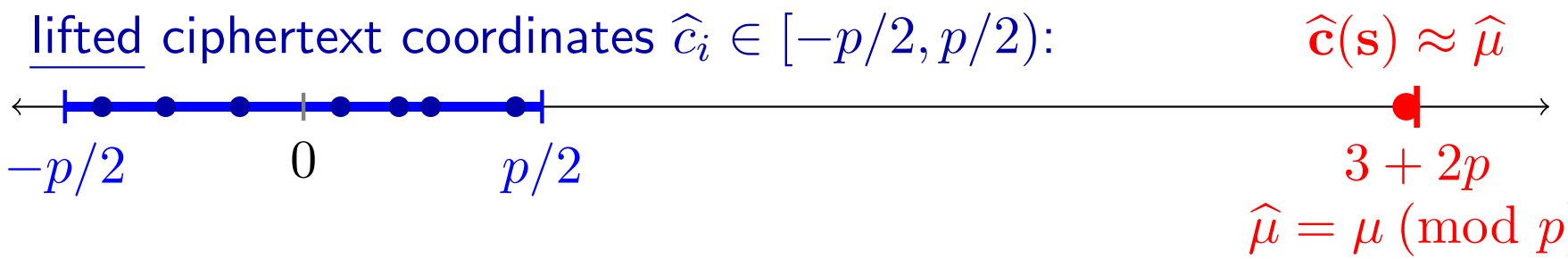

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Addition: trivial

Just add the ciphertext polynomials:

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Error blowup

Typically, $|\widehat{\mu}|, |\widehat{\mu}'| \approx p \cdot \|\mathbf{s}\|_2$, so multiplication amplifies noise by $\gg p$.

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- (Inspired by [CCKS'23] approx FHE, but technically very different.)

Plaintext Decomposition as a Ring Isomorphism

- Fix a base b and define the $\mathbb{Z}[B]$-ideal

$$\mathcal{L} := \{\mathbf{z}(B) \in \mathbb{Z}[B] : \mathbf{z}(b) = 0 \pmod{p}\} \supseteq p\mathbb{Z}.$$

("Gadget lattice": kernel of powers-of-b vector $(1, b, b^2, \ldots)$ [MicPei12])

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The Payoff

For $d = \lceil \log_b p \rceil$, every plaintext has many decompositions

$$\boldsymbol{\mu}(B) = \mu_0 + \mu_1 B + \cdots + \mu_{d-1} B^{d-1}, \qquad |\mu_i| \approx b \approx p^{1/d}.$$

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Toy example: let $b = 3$ and $p = b^2 = 9$.

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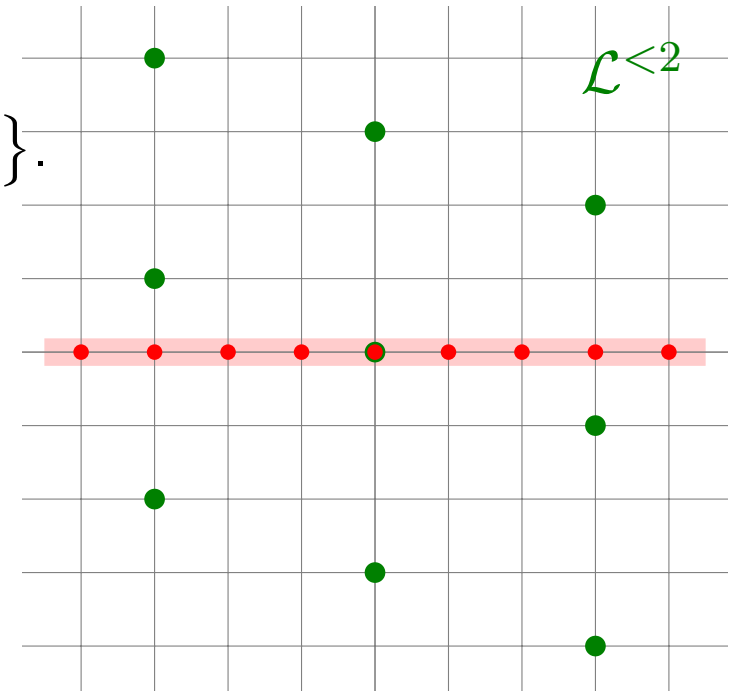

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In coefficient form, a short basis is

$$\mathbf{b}_1 = (3, -1), \qquad \mathbf{b}_2 = (0, 3).$$

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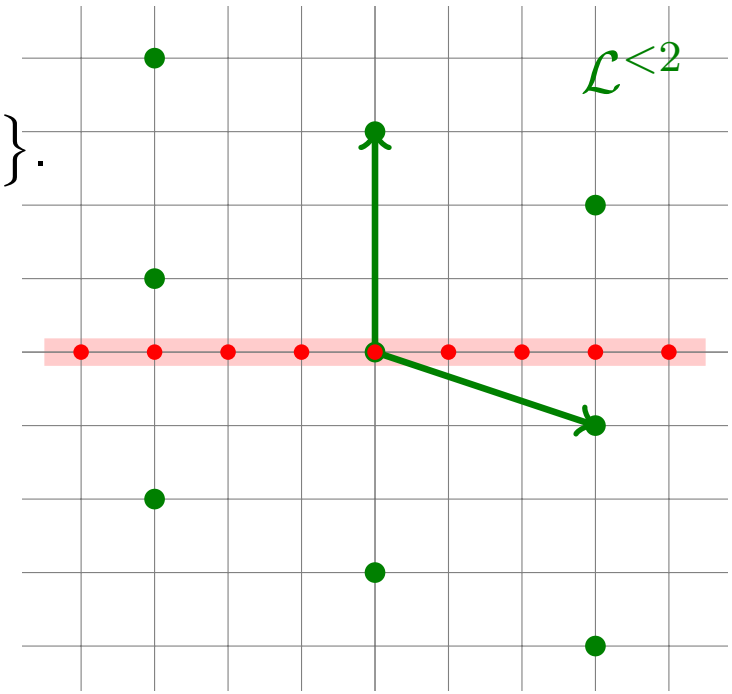

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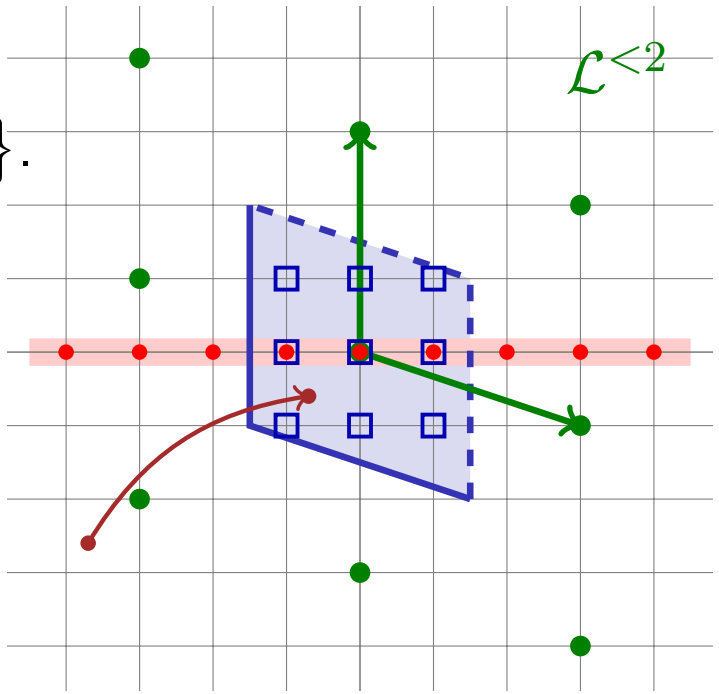
$$\mathcal{L}^{<2} := \left\{ z(B) \in \mathbb{Z}^{<2}[B] : z(3) = 0 \pmod 9 \right\}.$$

In coefficient form, a short basis is

$$\mathbf{b}_1 = (3, -1), \qquad \mathbf{b}_2 = (0, 3).$$



$\mathbb{Q}, \mathbb{Z}$ representatives mod $p\mathbb{Z}$

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- The decompositions can be noncanonical; only shortness is needed.

Decomposed BFV (dBFV)

A dBFV ciphertext encrypting $\mu \in \mathbb{Z}_p$ is a bivariate polynomial

$$\mathbf{c}(B, \vec{S}) \in (\mathbb{Q}[B]/\mathcal{L})[\vec{S}],$$

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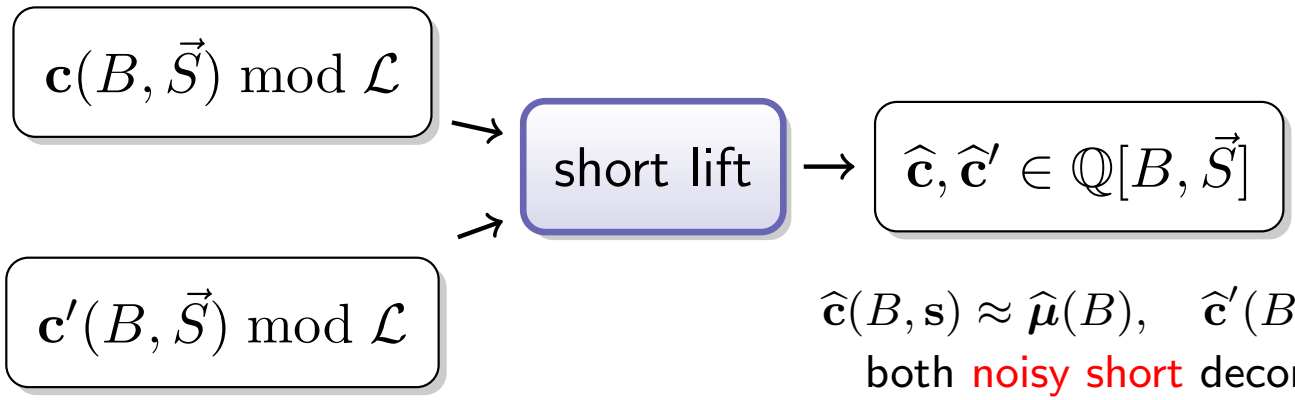
Fresh ciphertexts and security

- Fresh ciphertexts are mod-$\mathcal{L}$-LWE samples with error rate $\propto 1/p^{1/d}$.
- Hardness follows by a nearly lossless reduction from the corresponding standard plain/Module/Ring-LWE problem.

Homomorphic Multiplication in dBFV



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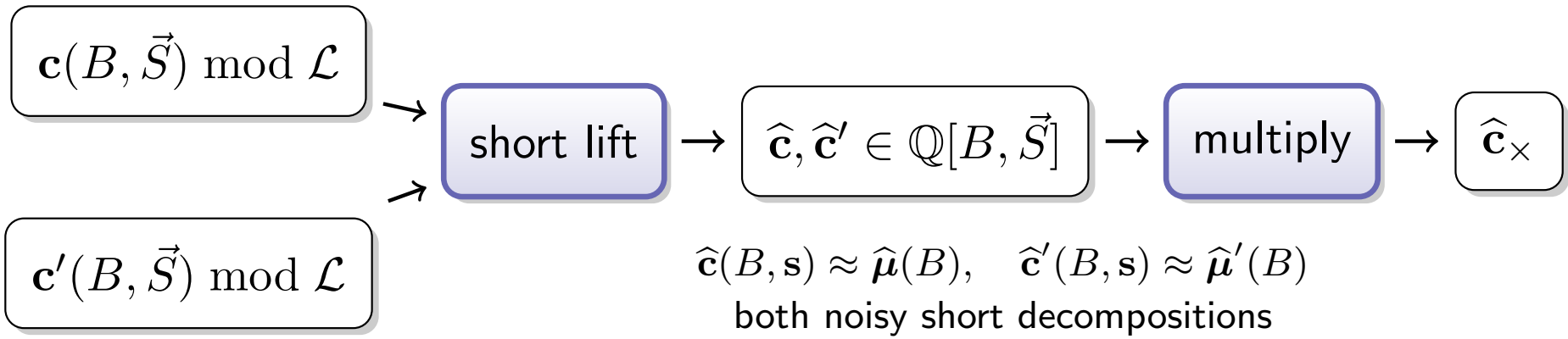

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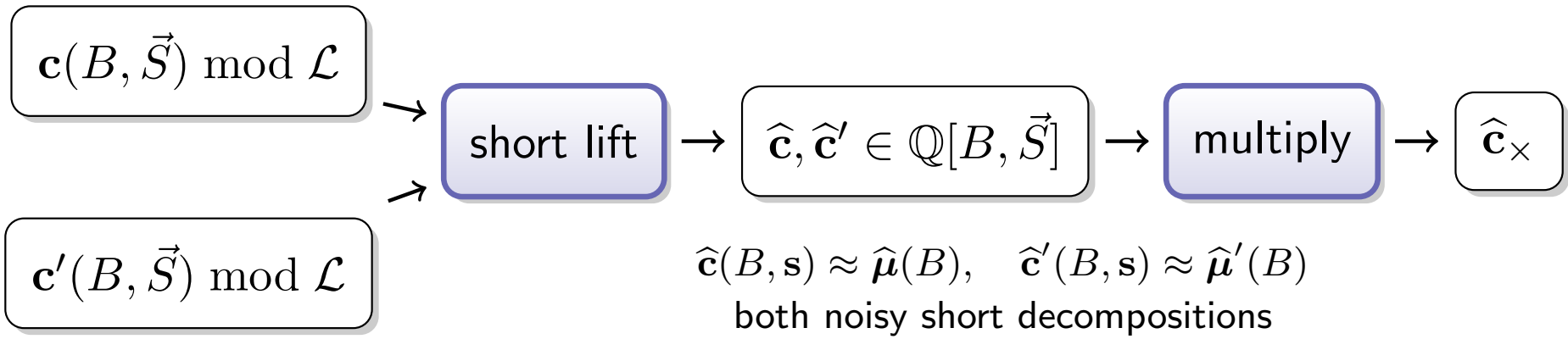

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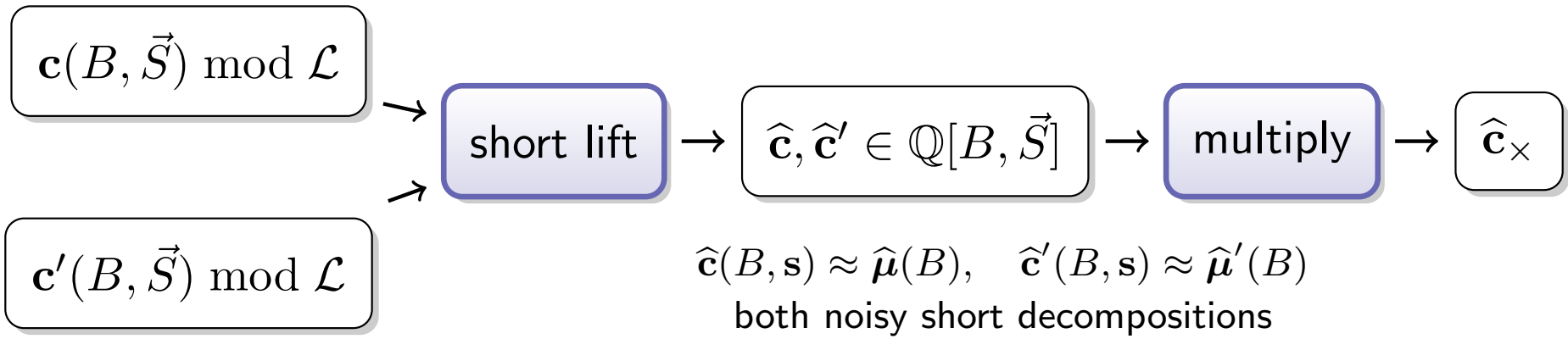

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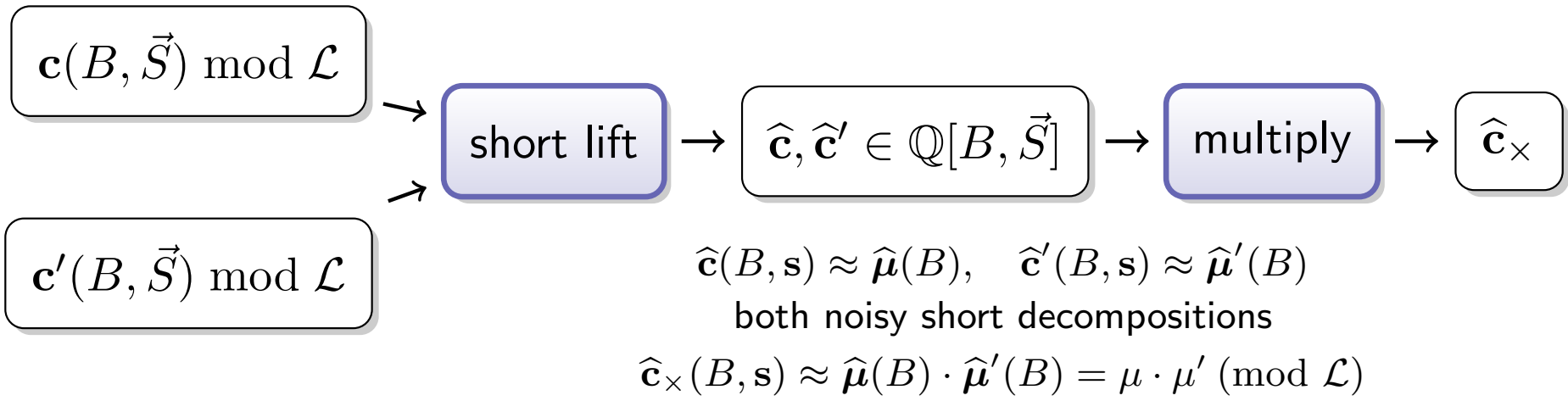

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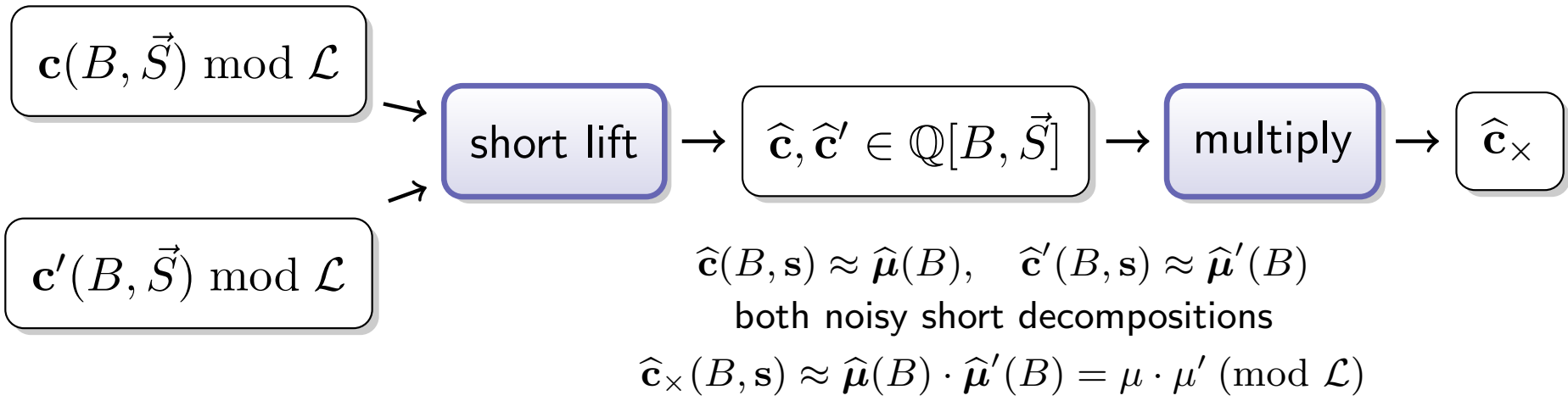

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Degree reduction in B

- For $p = b^d$, simply use $B^d = 0 \pmod{\mathcal{L}}$.
- In general, high powers of B reduce to short combos of low powers.

Putting It All Together

Using d-digit ciphertexts with base $b \approx p^{1/d}$:

	ordinary BFV	dBFV
Fresh LWE error rate	$\approx 1/p$	$\approx 1/p^{1/d}$
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- Compatibility with arbitrary moduli and rings, SIMD packing, and bootstrapping.

Direction II: Flexible SIMD Packing

Use subfields of cyclotomics

(Peikert–Pepin ePrint 2025/1760)

FHE (and Lattice Cryptography) over Rings

- Use rings (in number fields) for efficiency

[HPS'96,Mic'02,PR'06,LM'06,LPR'10,...]

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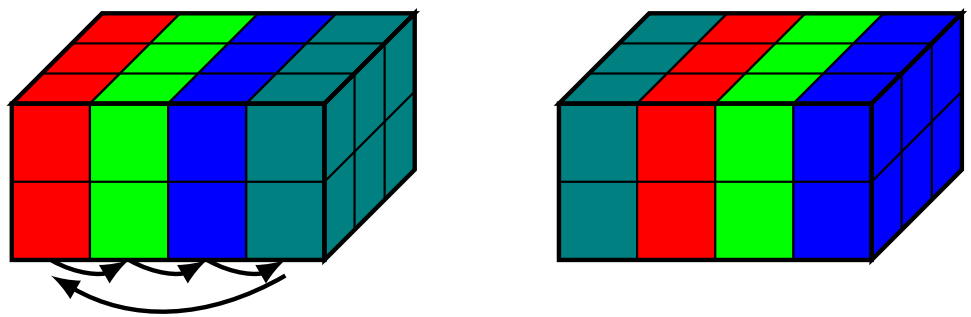
  - ★ certain vector permutations, for linear algebra and "packed bootstrapping" [BGV'12,GHS'12ab,HS'14]



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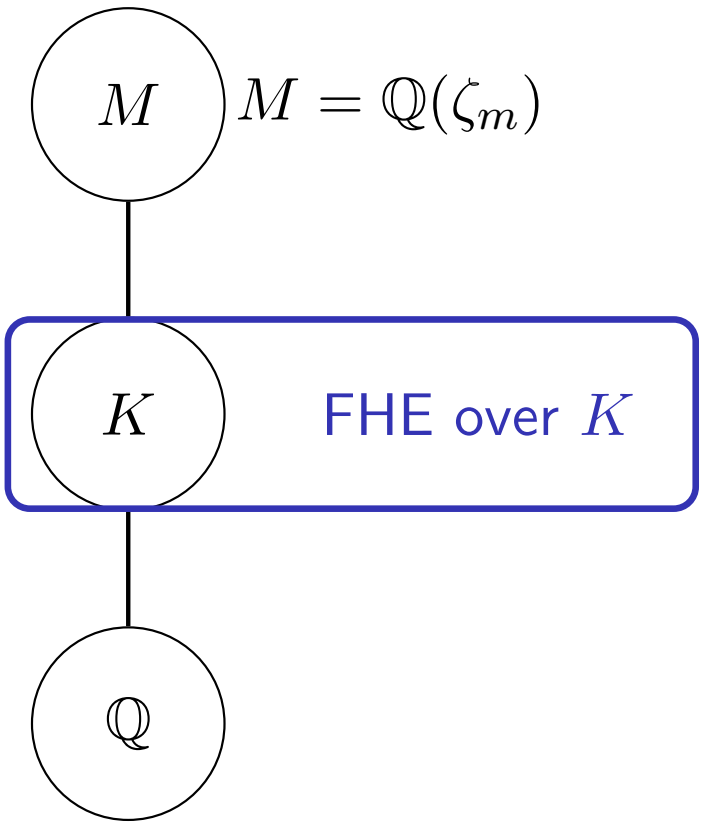
Use of cyclotomics entangles algebra, data layout, and security

Our Work: Beyond Cyclotomics



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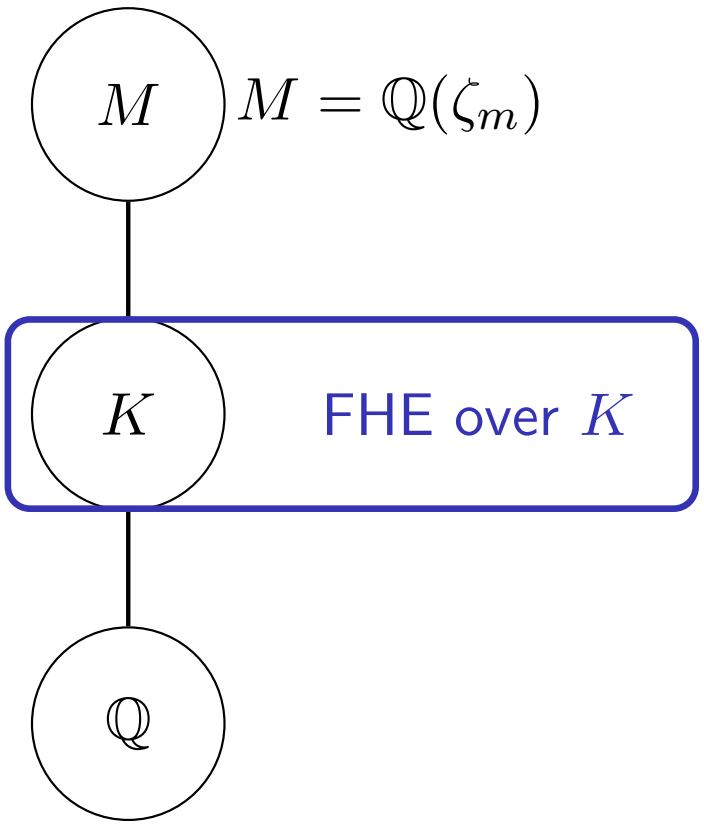

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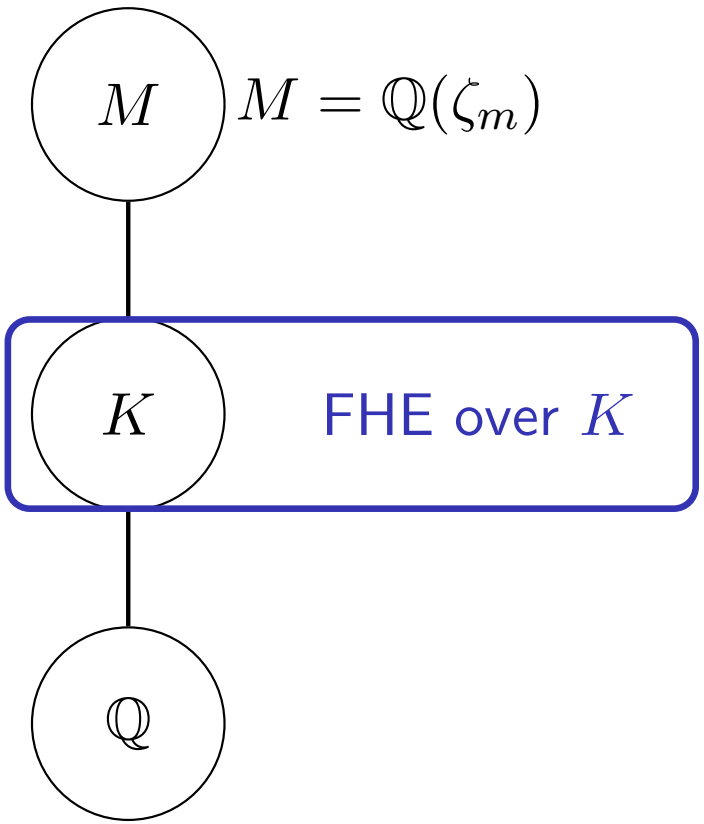

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Galois Theory Controls Slot Type and Count

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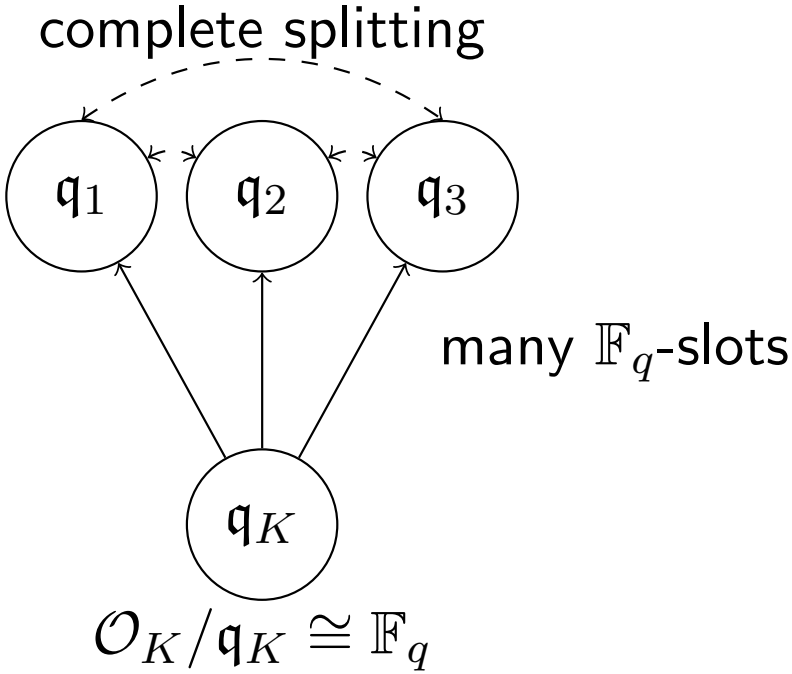

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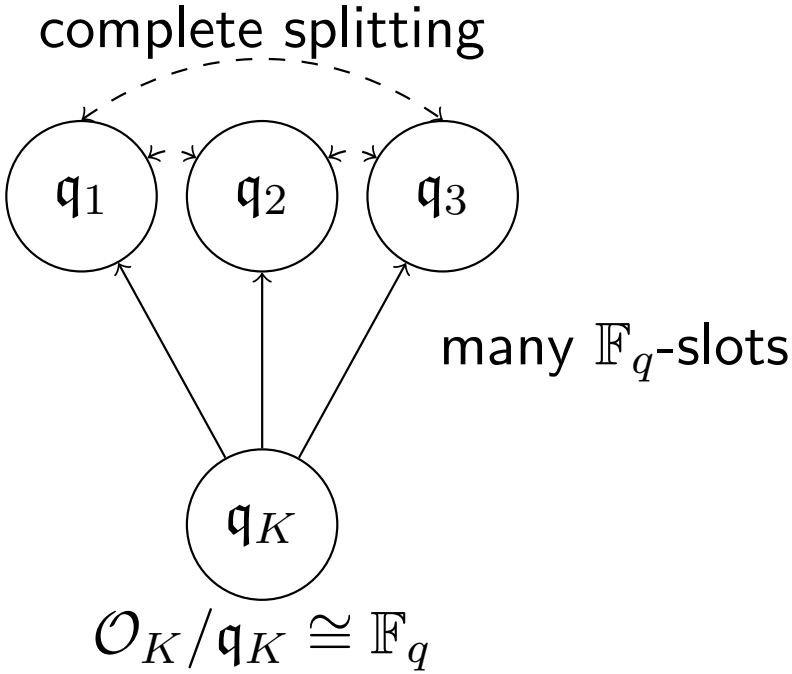

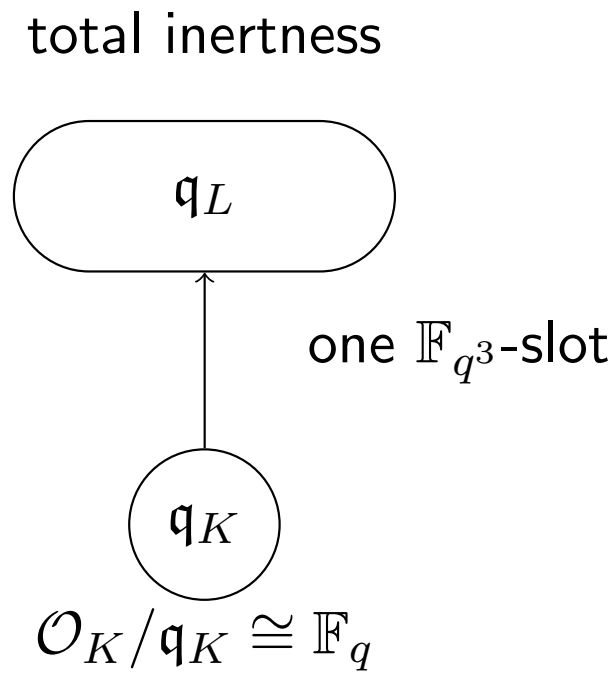

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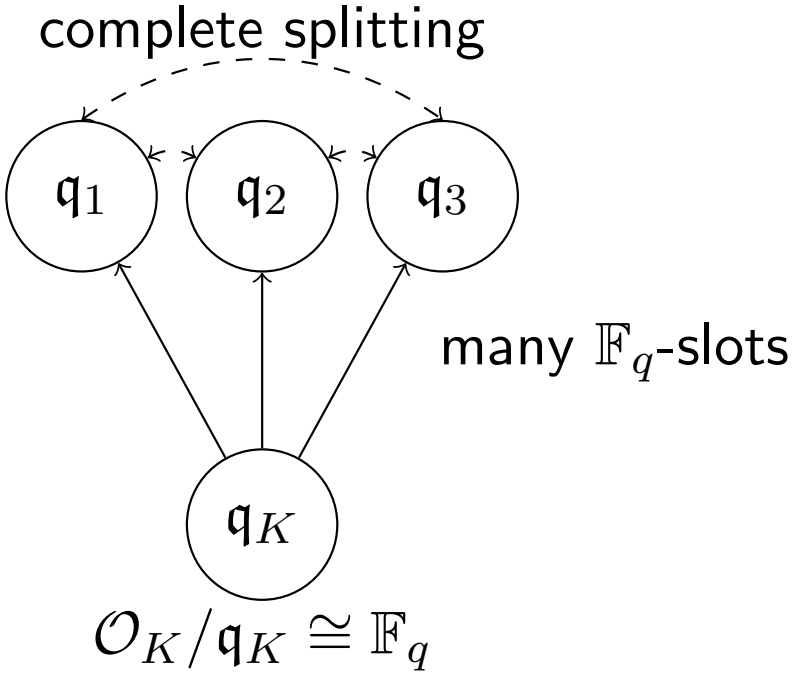

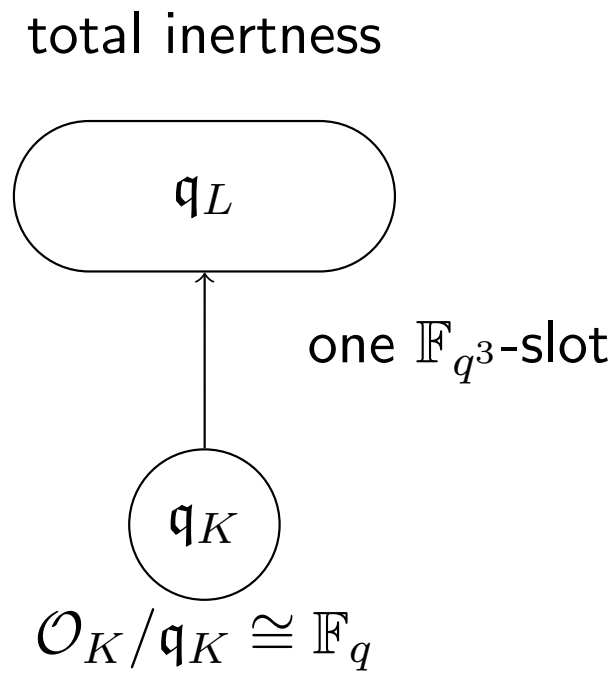

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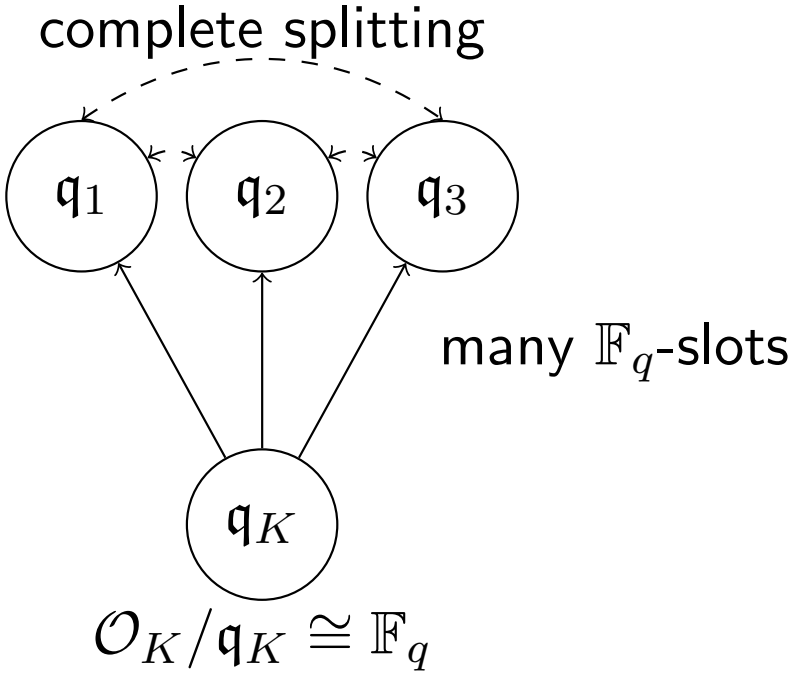

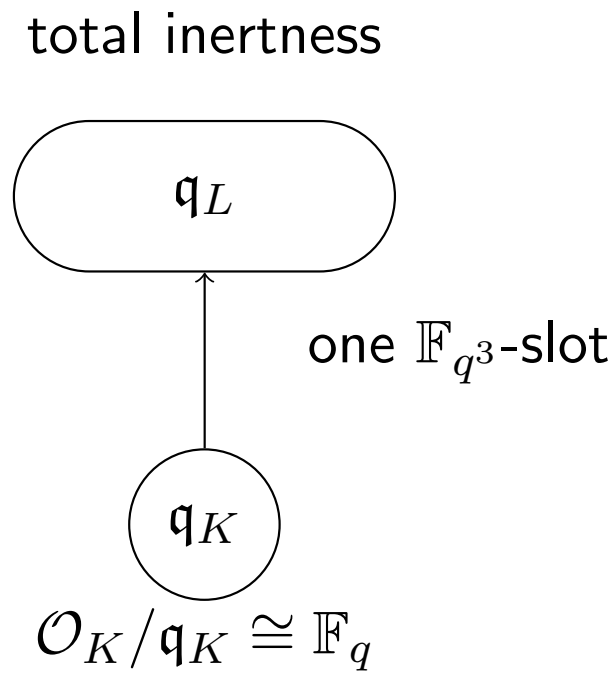

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Combine splitting and inertness across towers of subfields to get desired slot field and count.

Galois Factors Give the Slot-Tensor Shape and Movement

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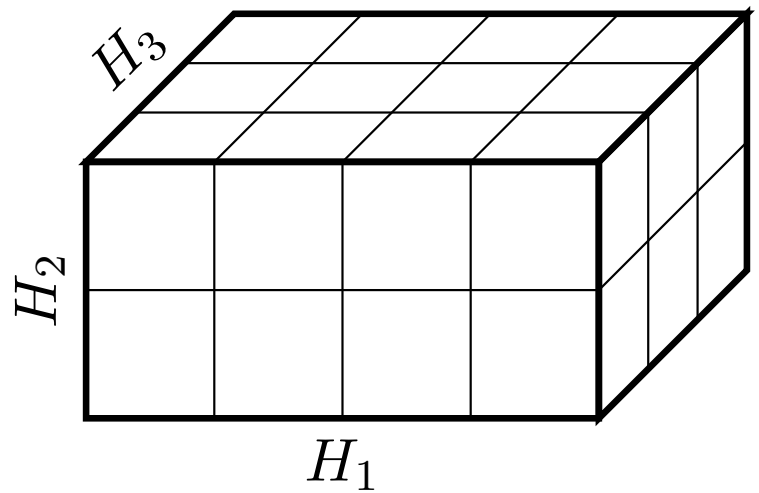

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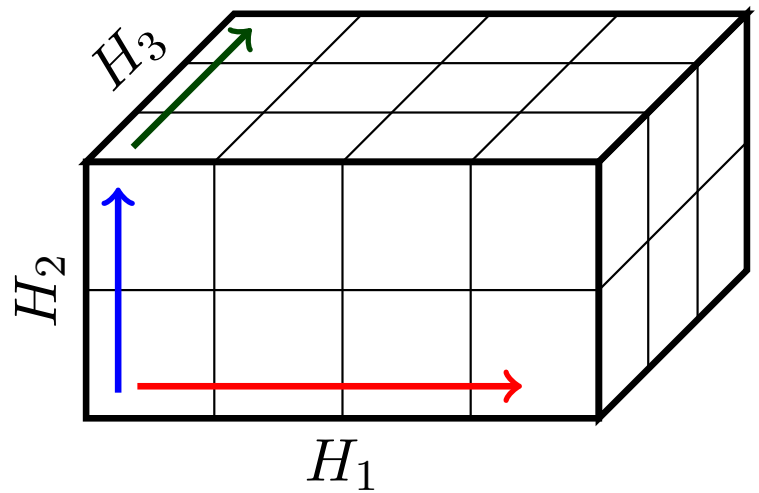

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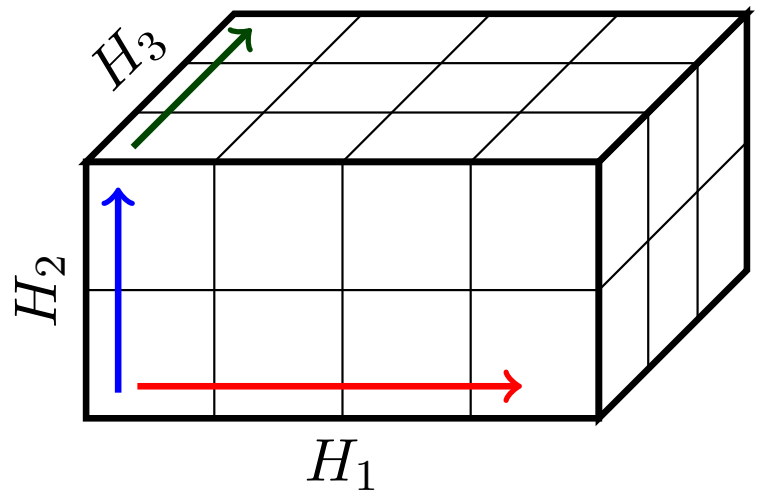

Automorphisms from each H_i move data along just one dimension.

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Bottom line

- Subfields add design freedom, and retain the machinery that make cyclotomics useful

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High precision: decompose the plaintext ring

$$\mathbb{Z}_p \cong \mathbb{Z}[B]/\mathcal{L} \quad \Longrightarrow \quad \text{noise growth } \approx d\, p^{1/d}.$$

Arbitrary modulus and ring; quasi-linear dependence on $\log p$.

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Thank you! Questions?